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Key Engineering Materials Vols. 183-187 (2000) pp. 499-504
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Stress Analysis of Toroidal Hole in an Infinite Body

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Keywords: Body Force Method, Elasticity, Numerical Analysis, Singular Integral Equation, Stress Concentration Factor, Toroidal Hole

ABSTRACT

This paper deals with stress analysis of toroidal hole in an infinite body under uniform tension. The problem is formulated as a system of singular integral equation with Cauchy-type singularity, where the densities of body forces distributing in the r- and z- directions are unknown functions. In order to satisfy the boundary conditions along the hole boundary, eight kinds of fundamental density functions are introduced in the similar way of previous papers treating plane stress problems. Then the body force densities are approximated by a linear combination of those fundamental density functions and polynomials. In the analysis, shape of toroidal hole is varied systematically; then, the magnitude and position of the maximum stress are shown in tables. The stress distributions along the boundary are shown in figures. The accuracy of the present analysis is verified by comparing the present results with the results obtained by the conventional method. It is found that this method gives rapid convergence numerical results for the stress distribution along the boundary and stress concentration factors of toroidal hole are close to stress concentration factors of notched round bar and deep notch when $a/d \to 1$.

1. Introduction

It is known that most engineering materials contain some defects in the form of holes, cavities and inclusions. To evaluate the effect of defects on the strength of structures, it is important to know the stress concentration of defects in a material. Therefore, a lot of useful results of 3-D stress concentration problems have been obtained by applying suitable numerical methods. For example, stress concentration problems of one and two spherical cavities were treated by several researchers [1-3]. However, there is few analyses for more than one ellipsoidal cavity is in a material. Because the degree of stress concentration depends on the shape, size, location of the ellipsoidal cavity, the

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loading conditions and other factors.

In preceding paper the authors have considered ellipsoidal cavities using singular integral equations of the body force method [4]. This method can be applied to the analysis of various shapes and spacing of ellipsoidal cavities. In this paper, the method will be applied to a stress concentration problem of toroidal hole. Then, stress concentration factor is calculated when the shape of toroidal hole is varied systematically. The stress concentration factor of toroidal hole is compared with stress concentration factor of notched round bar and deep notch.

2. Numerical Solution for Singular Integral Equations

Consider an infinite body having a toroidal hole under uniform z-direction tension as shown in Fig. 1. The problem is formulated in terms of singular integral equations, that is the stress field at an arbitrary point ($r=d+a\cos\psi, z=b\sin\psi$) when two ring forces act on another points ($\rho=d+a\cos\alpha, \zeta=\pm b\sin\alpha$) in an infinite body [5]. The formation is simply based on the principle of superposition. The integral equation is expressed by eq. (1), where the body force densities distributed along the prospective boundaries in the r-, z-directions are to be unknown functions.

$$
\begin{aligned}
&-(1/2)\{\rho_r^*(\psi)\cos\psi_0+\rho_z^*(\psi)\sin\psi_0\}+\int_{-\pi/2}^{\pi/2}K_{nn}^{Fr}(\alpha,\psi)\rho_r^*(\alpha)ds\\
&\qquad+\int_{-\pi/2}^{\pi/2}K_{nn}^{Fz}(\alpha,\psi)\rho_z^*(\alpha)ds=-\sigma_z^\infty\sin^2\psi_0\\
&-(1/2)\{-\rho_r^*(\psi)\sin\psi_0+\rho_z^*(\psi)\cos\psi_0\}+\int_{-\pi/2}^{\pi/2}K_{nt}^{Fr}(\alpha,\psi)\rho_r^*(\alpha)ds\\
&\qquad+\int_{-\pi/2}^{\pi/2}K_{nt}^{Fz}(\alpha,\psi)\rho_z^*(\alpha)ds=-\sigma_z^\infty\sin\psi_0\cos\psi_0
\end{aligned}
\tag{1}
$$

ψ_0 is an angle between the r-axis and the normal direction at the point (r,z). Equation (1) is virtually the boundary conditions at the imaginary boundary. The first terms of eq. (1) represent the stress due to the body force distributed on the ⊖ boundary. The ⊖ boundary means the imaginary boundary composed of the internal points that are infinitesimally apart from the initial boundary [6, 7].

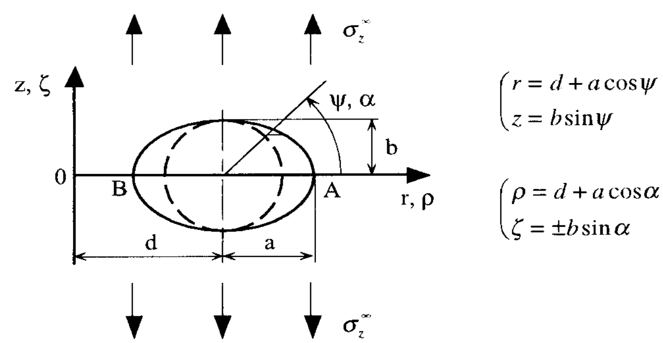

Fig. 1 Toroidal hole in an infinite body.

To solve eq. (1) is to determine the body force densities $\rho_r^*(\alpha), \rho_z^*(\alpha)$ in the region of $-\pi/2<\alpha<\pi/2$. Here, consider auxiliary functions $\rho_{r1}^*(\phi_k) \sim \rho_{r4}^*(\phi_k)$ and $\rho_{z1}^*(\phi_k) \sim \rho_{z4}^*(\phi_k)$ defined by eqs. (2), (3) instead of densities $\rho_r^*(\alpha), \rho_z^*(\alpha)$.

$$\rho_{r1}^*(\alpha) = \{\rho_r^*(\alpha) + \rho_r^*(\pi-\alpha) + \rho_r^*(\pi+\alpha) + \rho_r^*(-\alpha)\}/4 \quad (2.a)$$
$$\rho_{r2}^*(\alpha) = \{\rho_r^*(\alpha) + \rho_r^*(\pi-\alpha) - \rho_r^*(\pi+\alpha) - \rho_r^*(-\alpha)\}/4 \quad (2.b)$$
$$\rho_{r3}^*(\alpha) = \{\rho_r^*(\alpha) - \rho_r^*(\pi-\alpha) - \rho_r^*(\pi+\alpha) + \rho_r^*(-\alpha)\}/4 \quad (2.c)$$
$$\rho_{r4}^*(\alpha) = \{\rho_r^*(\alpha) - \rho_r^*(\pi-\alpha) + \rho_r^*(\pi+\alpha) - \rho_r^*(-\alpha)\}/4 \quad (2.d)$$
$$\rho_{z1}^*(\alpha) = \{\rho_z^*(\alpha) + \rho_z^*(\pi-\alpha) + \rho_z^*(\pi+\alpha) + \rho_z^*(-\alpha)\}/4 \quad (3.a)$$
$$\rho_{z2}^*(\alpha) = \{\rho_z^*(\alpha) + \rho_z^*(\pi-\alpha) - \rho_z^*(\pi+\alpha) - \rho_z^*(-\alpha)\}/4 \quad (3.b)$$
$$\rho_{z3}^*(\alpha) = \{\rho_z^*(\alpha) - \rho_z^*(\pi-\alpha) - \rho_z^*(\pi+\alpha) + \rho_z^*(-\alpha)\}/4 \quad (3.c)$$
$$\rho_{z4}^*(\alpha) = \{\rho_z^*(\alpha) - \rho_z^*(\pi-\alpha) + \rho_z^*(\pi+\alpha) - \rho_z^*(-\alpha)\}/4 \quad (3.d)$$

These new functions $\rho_{r1}^*(\phi_k) \sim \rho_{r4}^*(\phi_k)$ and $\rho_{z1}^*(\phi_k) \sim \rho_{z4}^*(\phi_k)$ must satisfy eqs. (4), (5) because of the definition (4), (5).

$$\rho_{r1}^*(\alpha) = \rho_{r1}^*(\pi-\alpha) = \rho_{r1}^*(\pi+\alpha) = \rho_{r1}^*(-\alpha) \quad (4.a)$$
$$\rho_{r2}^*(\alpha) = \rho_{r2}^*(\pi-\alpha) = -\rho_{r2}^*(\pi+\alpha) = -\rho_{r2}^*(-\alpha) \quad (4.b)$$
$$\rho_{r3}^*(\alpha) = -\rho_{r3}^*(\pi-\alpha) = -\rho_{r3}^*(\pi+\alpha) = \rho_{r3}^*(-\alpha) \quad (4.c)$$
$$\rho_{r4}^*(\alpha) = -\rho_{r4}^*(\pi-\alpha) = \rho_{r4}^*(\pi+\alpha) = -\rho_{r4}^*(-\alpha) \quad (4.d)$$
$$\rho_{z1}^*(\alpha) = \rho_{z1}^*(\pi-\alpha) = \rho_{z1}^*(\pi+\alpha) = \rho_{z1}^*(-\alpha) \quad (5.a)$$
$$\rho_{z2}^*(\alpha) = \rho_{z2}^*(\pi-\alpha) = -\rho_{z2}^*(\pi+\alpha) = -\rho_{z2}^*(-\alpha) \quad (5.b)$$
$$\rho_{z3}^*(\alpha) = -\rho_{z3}^*(\pi-\alpha) = -\rho_{z3}^*(\pi+\alpha) = \rho_{z3}^*(-\alpha) \quad (5.c)$$
$$\rho_{z4}^*(\alpha) = -\rho_{z4}^*(\pi-\alpha) = \rho_{z4}^*(\pi+\alpha) = -\rho_{z4}^*(-\alpha) \quad (5.d)$$

It should be noted that determining auxiliary functions $\rho_{r1}^*(\phi_k) \sim \rho_{z4}^*(\phi_k)$ in the range $0<\alpha<\pi/2$ is equivalent to determining original unknown densities $\rho_r^*(\alpha)$, $\rho_z^*(\alpha)$ in the range $-\pi/2<\alpha<\pi/2$. In other words, if the auxiliary functions $\rho_{r1}^*(\phi_k) \sim \rho_{z4}^*(\phi_k)$ are given in the range $0<\alpha<\pi/2$, original unknown functions $\rho_r^*(\alpha)$, $\rho_z^*(\alpha)$ are expressed in the range $-\pi/2<\alpha<\pi/2$ using eqs. (6), (7).

$$\rho_r^*(\alpha) = \rho_{r1}^*(\alpha) + \rho_{r2}^*(\alpha) + \rho_{r3}^*(\alpha) + \rho_{r4}^*(\alpha) \quad (6.a)$$
$$\rho_r^*(\pi-\alpha) = \rho_{r1}^*(\alpha) + \rho_{r2}^*(\alpha) - \rho_{r3}^*(\alpha) - \rho_{r4}^*(\alpha) \quad (6.b)$$
$$\rho_r^*(\pi+\alpha) = \rho_{r1}^*(\alpha) - \rho_{r2}^*(\alpha) - \rho_{r3}^*(\alpha) + \rho_{r4}^*(\alpha) \quad (6.c)$$
$$\rho_r^*(-\alpha) = \rho_{r1}^*(\alpha) - \rho_{r2}^*(\alpha) + \rho_{r3}^*(\alpha) - \rho_{r4}^*(\alpha) \quad (6.d)$$
$$\rho_z^*(\alpha) = \rho_{z1}^*(\alpha) + \rho_{z2}^*(\alpha) + \rho_{z3}^*(\alpha) + \rho_{z4}^*(\alpha) \quad (7.a)$$
$$\rho_z^*(\pi-\alpha) = \rho_{z1}^*(\alpha) + \rho_{z2}^*(\alpha) - \rho_{z3}^*(\alpha) - \rho_{z4}^*(\alpha) \quad (7.b)$$
$$\rho_z^*(\pi+\alpha) = \rho_{z1}^*(\alpha) - \rho_{z2}^*(\alpha) - \rho_{z3}^*(\alpha) + \rho_{z4}^*(\alpha) \quad (7.c)$$
$$\rho_z^*(-\alpha) = \rho_{z1}^*(\alpha) - \rho_{z2}^*(\alpha) + \rho_{z3}^*(\alpha) - \rho_{z4}^*(\alpha) \quad (7.d)$$

The fundamental density functions for the body forces in the r-, and z-directions are defined by following expression:

$$w_{r1}(\alpha) = n_r(\alpha)/\cos\alpha, w_{r2}(\alpha) = n_r(\alpha)\tan\alpha, w_{r3}(\alpha) = n_r(\alpha), w_{r4}(\alpha) = n_r(\alpha)\sin\alpha \quad (8)$$
$$w_{z1}(\alpha) = n_z(\alpha)/\sin\alpha, w_{z2}(\alpha) = n_z(\alpha), w_{z3}(\alpha) = n_z(\alpha)\cot\alpha, w_{z4}(\alpha) = n_z(\alpha)\cos\alpha \quad (9)$$

Using eqs. (8), (9), original body force densities are expressed as shown in eqs. (10), (11).

$$\rho_{rj}^*(\alpha)=\rho_{rj}(\alpha)w_{rj}(\alpha) \qquad j=1,2,3,4 \tag{10}$$

$$\rho_{zj}^*(\alpha)=\rho_{zj}(\alpha)w_{zj}(\alpha) \qquad j=1,2,3,4 \tag{11}$$

where $\rho_{r1}(\alpha)$~$\rho_{r4}(\alpha)$ and $\rho_{z1}(\alpha)$~$\rho_{z4}(\alpha)$ are unknown functions, which have been called weight functions. Then all $\rho_{r1}(\alpha)$~$\rho_{r4}(\alpha)$ and $\rho_{z1}(\alpha)$~$\rho_{z4}(\alpha)$ must satisfy eq. (12).

$$f(\alpha)=f(\pi-\alpha)=f(\pi+\alpha)=f(-\alpha)f(\phi_k) : \rho_{r1}(\alpha)\sim\rho_{z4}(\alpha) \tag{12}$$

Finally, original unknown densities $\rho_r^*(\alpha), \rho_z^*(\alpha)$ can be expressed in eq. (13) as linear combination of the fundamental densities and the weight functions.

$$\rho_r^*(\alpha)=\sum_{j=1}^{4}\rho_{rj}(\alpha)w_{rj}(\alpha), \rho_z^*(\alpha)=\sum_{j=1}^{4}\rho_{zj}(\alpha)w_{zj}(\alpha) \tag{13}$$

By considering the symmetry of the problem, $w_{r1}(\alpha)$, $w_{r3}(\alpha)$, $w_{z2}(\alpha)$, $w_{z4}(\alpha)$ are suitable. Unknown functions $\rho_r^*(\alpha)$, $\rho_z^*(\alpha)$ can be expressed by the following equation.

$$\rho_r^*(\alpha)=\rho_{r1}(\alpha)w_{r1}(\alpha)+\rho_{r3}(\alpha)w_{r3}(\alpha), \rho_z^*(\alpha)=\rho_{z2}(\alpha)w_{z2}(\alpha)+\rho_{z4}(\alpha)w_{z4}(\alpha) \tag{14}$$

Here all unknown weight functions can be approximated as shown in eqs. (15), (16) because all of these must satisfy eq. (13).

$$\rho_{r1}(\alpha)=\sum_{n=1}^{M/2}a_n t_n(\alpha), \rho_{r3}(\alpha)=\sum_{n=1}^{M/2}b_n t_n(\alpha), \rho_{z2}(\alpha)=\sum_{n=1}^{M/2}c_n t_n(\alpha), \rho_{z4}(\alpha)=\sum_{n=1}^{M/2}d_n t_n(\alpha) \tag{15}$$

$$t_n(\alpha)=\cos\{2(n-1)\alpha\} \tag{16}$$

Where M is the number of the collocation points in $0<\alpha<2\pi$. Using the approximation method mentioned above, we can obtain the system of linear equations for determining the coefficients a_n, b_n, c_n, d_n. Then, the magnitude and position of the maximum stress are calculated, when the shape of the toroidal hole is changed systematically.

3. Numerical Results and Discussion

Table 1 shows the convergence of the values of stress σ_n, σ_t, τ_{nt} along the toroidal hole boundary with increasing the collocation number M when a/b=1, a/d=2/3, ν=0.3, $\sigma_z^\infty=1$ in Fig. 1. In the present analysis, the boundary conditions (σ_n=0, τ_{nt}=0), which should be zero along the boundary, are less than 10^{-5} even when M=16. Therefore, the boundary requirements can be highly satisfied along the entire boundary.

In Table 2 the convergence of the stress concentration factors at point A ($\psi=0^\circ$) and B ($\psi=180^\circ$) is shown to be compared with the conventional body force method using step-function when a/b=1, a/d=0.9, ν=0.3, $\sigma_z^\infty=1$ in Fig. 1. In table 2, the symbol ∞ designates the extrapolated value using the results M=32 and 48. The present results show rapid convergence than the results using the step-function which need the extrapolation.

In Table 3 the stress concentration factor is shown to be compared with the conventional body force method using step-function when a/b=1, ν=0.3, $\sigma_z^\infty=1$ in Fig. 1. The solution of the notched round bar [5, 8] and the deep notch [9] are also shown in Table 3 for reference. The present results coincide with the results of the conventional body force method in the 4 digits. The stress concentration factors of toroidal hole are close to stress concentration factors of notched round bar and deep notch when a/d→1.

Table 1 Stress distribution along the boundary when a/b=1, a/d=2/3, ν=0.3, $\sigma_z^\infty = 1$ in Fig. 1.

ψ (deg.)	M	σ_t	σ_n	τ_{nt}
0	8	2.69383	1.0×10^{-5}	0
	12	2.69676	2.3×10^{-5}	0
	16	2.69373	9.5×10^{-7}	0
	20	2.69373	-3.9×10^{-7}	0
40	8	1.37277	3.8×10^{-4}	3.9×10^{-4}
	12	1.37306	-1.7×10^{-5}	-2.8×10^{-6}
	16	1.37307	2.0×10^{-7}	-1.2×10^{-8}
	20	1.37307	-3.6×10^{-8}	2.8×10^{-10}
80	8	-0.62466	2.6×10^{-4}	6.9×10^{-5}
	12	-0.62409	-2.9×10^{-5}	6.6×10^{-6}
	16	-0.62408	-1.5×10^{-6}	7.4×10^{-7}
	20	-0.62408	-3.7×10^{-8}	2.1×10^{-9}
100	8	-0.76442	3.5×10^{-4}	-5.0×10^{-6}
	12	-0.76488	-3.3×10^{-5}	1.8×10^{-5}
	16	-0.76478	-1.5×10^{-6}	1.3×10^{-6}
	20	-0.76478	-3.1×10^{-8}	5.4×10^{-8}
140	8	1.65287	1.8×10^{-3}	-1.1×10^{-3}
	12	1.65293	-1.7×10^{-5}	4.3×10^{-5}
	16	1.6528	3.9×10^{-8}	-4.6×10^{-7}
	20	1.65281	1.3×10^{-9}	-9.2×10^{-9}
180	8	4.30702	1.2×10^{-3}	0
	12	4.30589	-3.7×10^{-5}	0
	16	4.30593	-2.1×10^{-6}	0
	20	4.30593	-9.5×10^{-8}	0

Table 2 Convergence of the maximum stress when a/b=1, a/d=0.9, ν=0.3, $\sigma_z^\infty = 1$ in Fig. 1.

	Present analysis			Step-function (ρ_{r3}, ρ_{z2})		
M	K_{tA}	K_{tB}	K_t	K_{tA}	K_{tB}	K_t
4	2.7032	8.3236	1.0242	2.6894	8.9249	1.0185
8	2.6529	8.4751	1.0422	2.6446	8.8165	1.0330
12	2.6373	8.3918	1.0470	2.6387	8.5311	1.0403
16	2.6343	8.3802	1.0476	2.6368	8.4558	1.0435
20	2.6338	8.3785	1.0477	2.6353	8.4132	1.0458
24	2.6337	8.3783	1.0477	2.6347	8.3989	1.0465
28	2.6336	8.3783	1.0477	2.6343	8.3884	1.0470
∞				2.633	8.373	1.048

Table 3 Stress Concentration factor for toroidal hole when a/b=1, ν=0.3, $\sigma_z^{\infty} = 1$ in Fig. 1.
[$K_t = \sigma_{zB}/\sigma_n$, $\sigma_n = F/\{\pi(d-a)^2\}$]

	Present analysis	Step-function (ρ_{r3}, ρ_{z2})	Notched round bar		Deep notch
a/d	K_t	K_t	K_t [5]	K_t [8]	K_t [9]
0	3.0000	3.0000	3.065	3.065	∞
0.1	2.5484	2.5484	2.601	2.593	3.1845
0.2	2.1836	2.1836	2.196	2.191	2.2272
0.3	1.8803	1.8803	1.869	1.871	1.8052
0.4	1.6309	1.6308	1.610	1.608	1.5571
0.5	1.4321	1.4320	1.412	1.411	1.3908
0.6	1.2820	1.2818	1.270	1.270	1.2705
0.7	1.1762	1.1761	1.172	1.172	1.1790
0.8	1.1036	1.1037	1.103	1.101	1.1069
0.9	1.0477	1.0481	1.048	1.046	1.0484

4. Conclusions

In this paper, the numerical solution of the singular integral equations based on the body force method in toroidal hole problem was investigated. The conclusions were summarized as follows:

(1) The stress concentration problem of toroidal hole was formulated in terms of singular integral equations of the body force method. The unknown functions were approximated by the product of the fundamental density functions and polynomials.

(2) The accuracy of the present analysis was verified through examining the boundary conditions and the convergence of the maximum stress. The present results could highly satisfy the boundary conditions and showed rapid convergence than the conventional body force method.

(3) The stress concentration factors of toroidal hole were close to stress concentration factors of notched round bar and deep notch when a/d→1.

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